Continuous and Discrete Time: How Differing Perspectives on Modeling Time Affect Developmental Inferences

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Introduction

- Methodology for modeling repeated observations
- Availability of intraindividual data
- Intraindividual variability

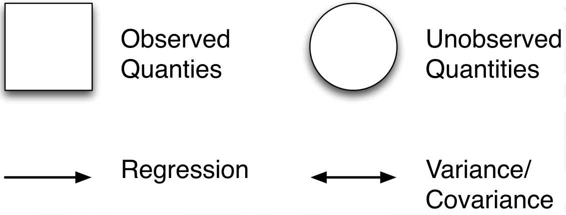
- Time is unlike other dimensions sampled in the social, behavioral, and medical sciences

- Sampling across time is always discrete, the underlying dimension is continuous

Today's Presentation

- Provide two Perspectives: Discrete, Continuous Time
- Contrasting Two Models: Implicit, Explicit Consideration of Time Between Observations
- Substantive Example: Similarities and Differences in Inferences
- Challenges and Potential of Continuous Time Models

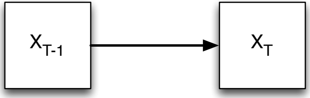
Diagramning Conventions



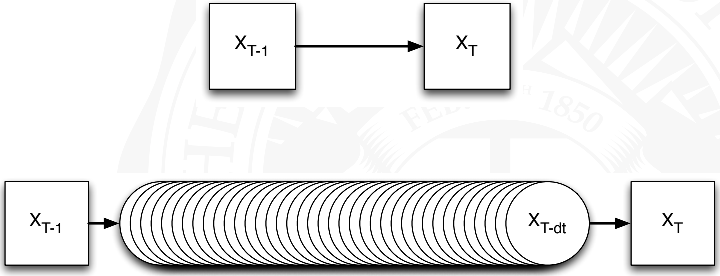
Note: While using conventions common to Structural Equation Modeling, many diagrams in the presentation will neglect variances, covariances to focus on key conceptual issues

Repeated Observations

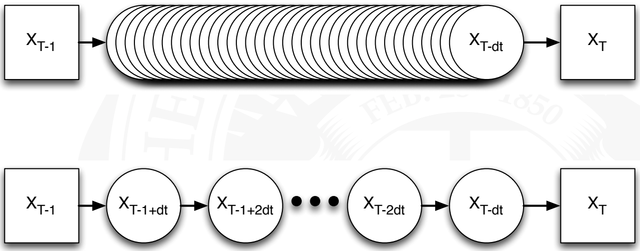
Repeated Observations



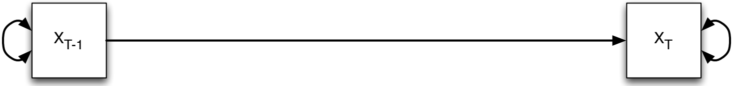
The Fundamental Difference



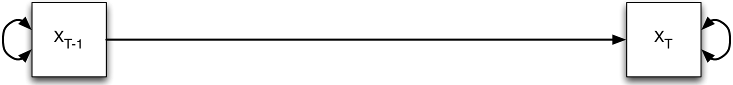
Diagramming Conventions



Implicit Modeling of Time



Implicit Modeling of Time

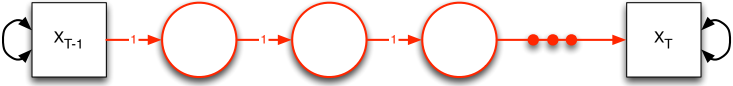


- $X_T = \beta_0 + \beta_1 X_{T-1}$
- Implicit Time, Discrete Time Model

How do People Change from One Time to the Next?

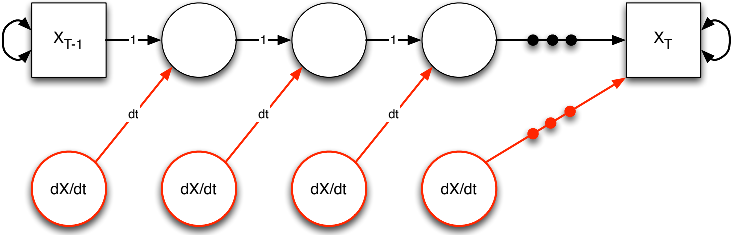
- $X_T =$ some function of time
- Assumption: the constructs exist between A and B

The Construct Exists between A and B



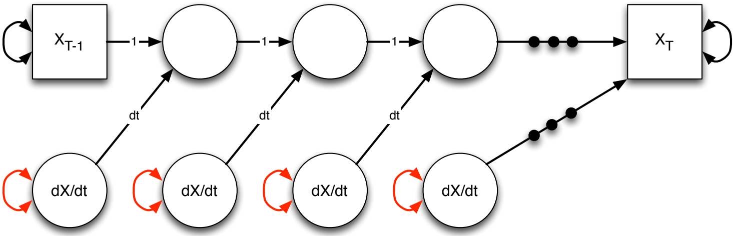
- Silly model if interested in development
- $X_T = 1 * X_{T-dt}$
- dt is some infinitesimally small amount of time
- Does the construct change?

The Construct Changes



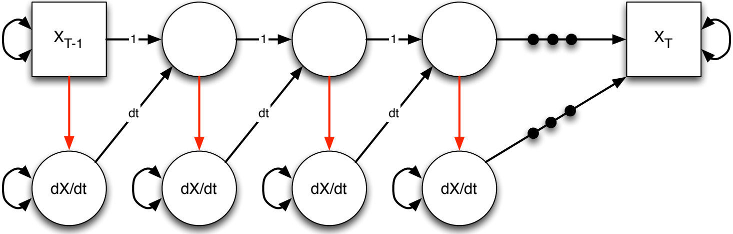
- $X_T = 1 * X_{T-dt} + (dt) * \left(\frac{dX}{dt}\right)$
- $\frac{dX}{dt}$ is the momentary rate of change of X with respect to time (first derivative/velocity)
- $X_{current} = X_{prior} + (\text{time changing})(\text{rate of change})$
- What defines the change from one moment to the next?

There could be Stochastic (Random) Perturbations



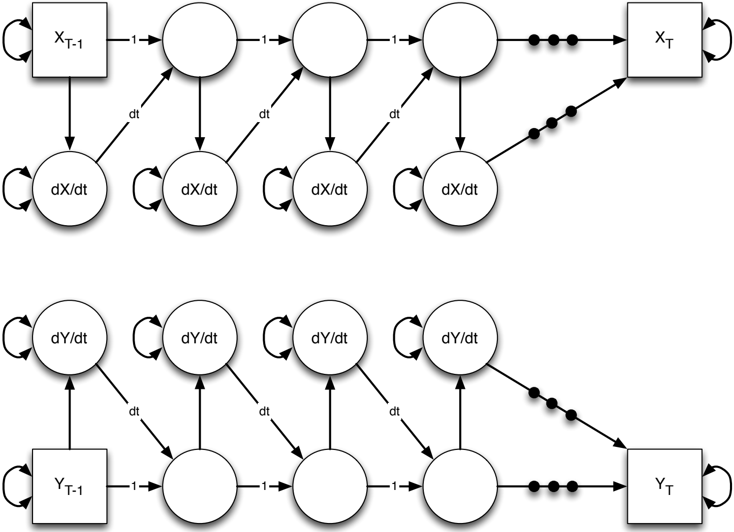
- $X_T = 1 * X_{T-dt} + (dt) * \left(\frac{dX}{dt}\right)$
- $\frac{dX}{dt} = \varepsilon_{dt}$
- $\varepsilon_{dt} \sim N(0, \sigma^2)$

Constructs May Self–Regulate



- $X_T = 1 * X_{T-dt} + (dt) * \left(\frac{dX}{dt}\right)$
- $\frac{dX}{dt} = \beta X_{T-dt} + \varepsilon_{dt}$
- $\varepsilon_{dt} \sim N(0, \sigma^2)$

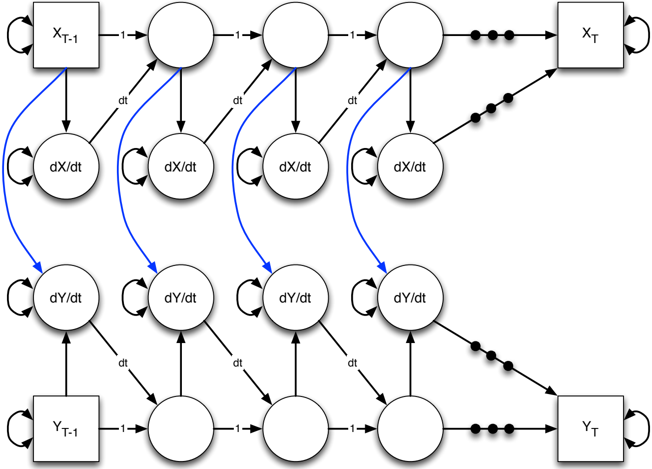
Do Constructs Affect Each Other? How?



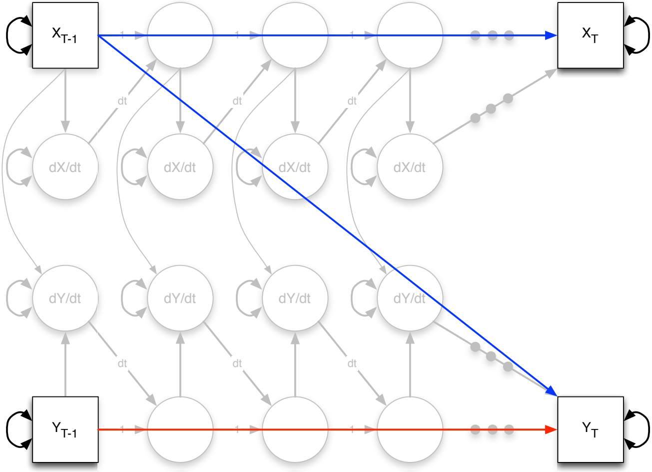
How Do Variables Affect Each Other?

		Construct 2	
		Level	**Velocity**
Construct 1	**Level**	Level-Level: Are high *levels* of maternal depression observed with high *levels* of child behavior problem?	
	Velocity	Velocity-Level: Is a mother's *level* of depression, regardless of whether her symptoms are changing or not, related to the rate at which her child's behavior problems increase or decrease (*velocity*)?	Velocity-Velocity: Does the rate at which mother's depressive symptoms increase or decrease (*velocity)* predict the rate at which her child's behavior problems increase or decrease (*velocity*), independent of her level of depression?

Perhaps Level Relates to Velocity (Change)



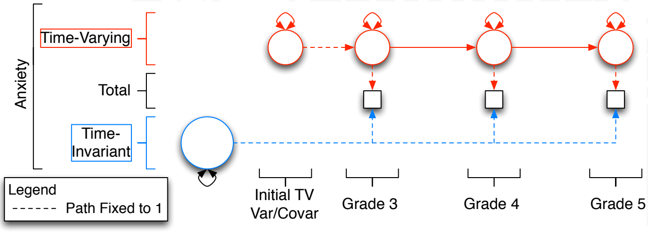
Implicit Time Modeling



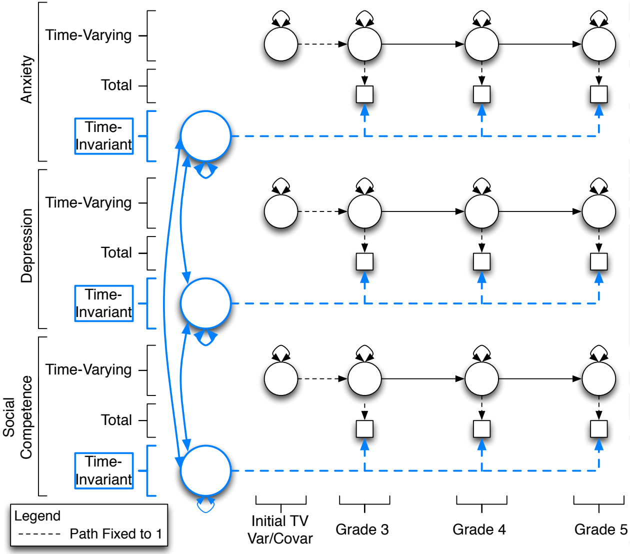
Substantive Example

- Cole, Martin, Peeke, Seroczynski, & Fier, 1999; Cole, Martin, & Powers, 1997; Cole, Peeke, Dolezal, Murray, & Canzoniero, 1999
- 291 children in grades 3, 4, and 5
- Children were assessed on Anxiety, Depression, and Social Competence (self-report of the children)
- Model are built on literature suggesting that both Anxiety (X) and Depression (M) affect Social Competence (Y), but that Anxiety (X) is often a precursor to Depression (M)
- Longitudinal Mediation

Discrete Time Model: Cross-lagged Panel Model



Discrete Time Model: Cross-lagged Panel Model



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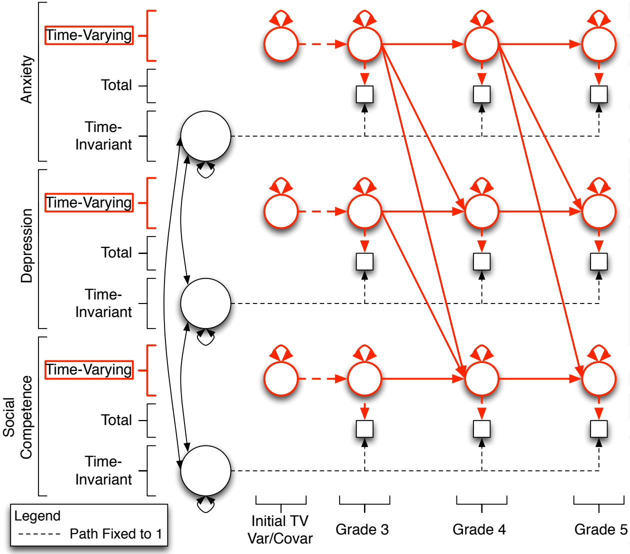

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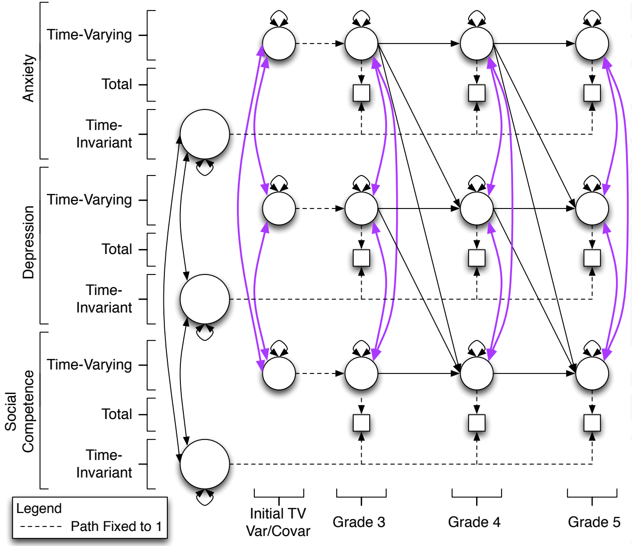

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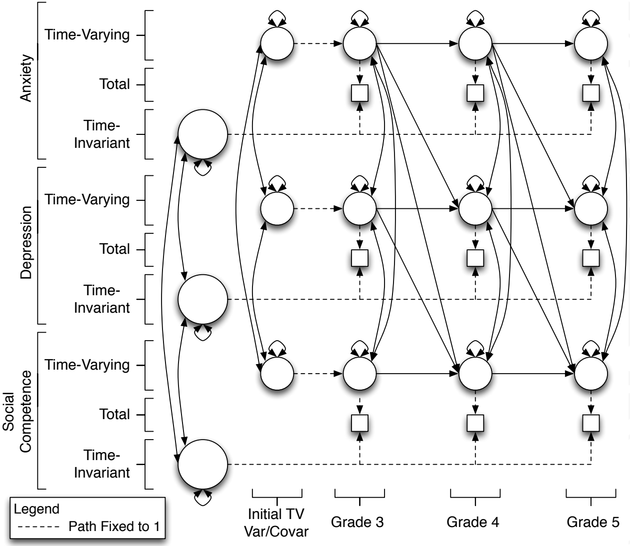
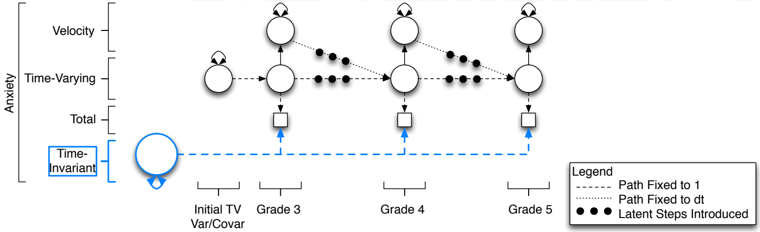
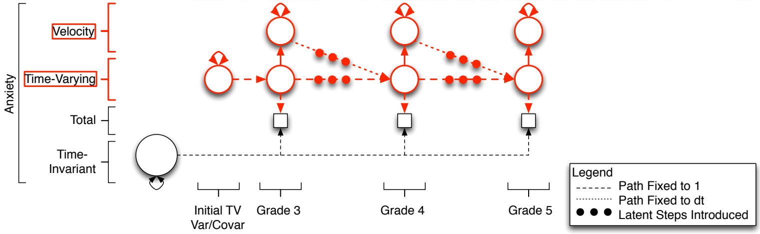


Continuous Time Model



Continuous Time Model



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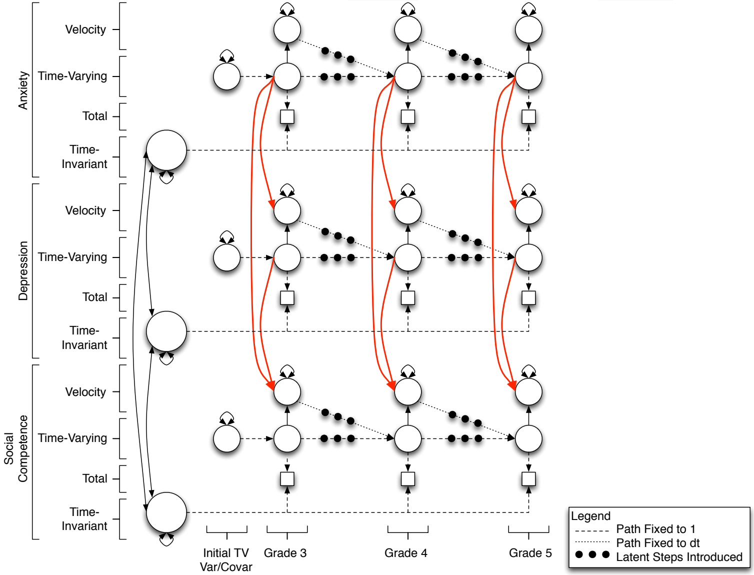
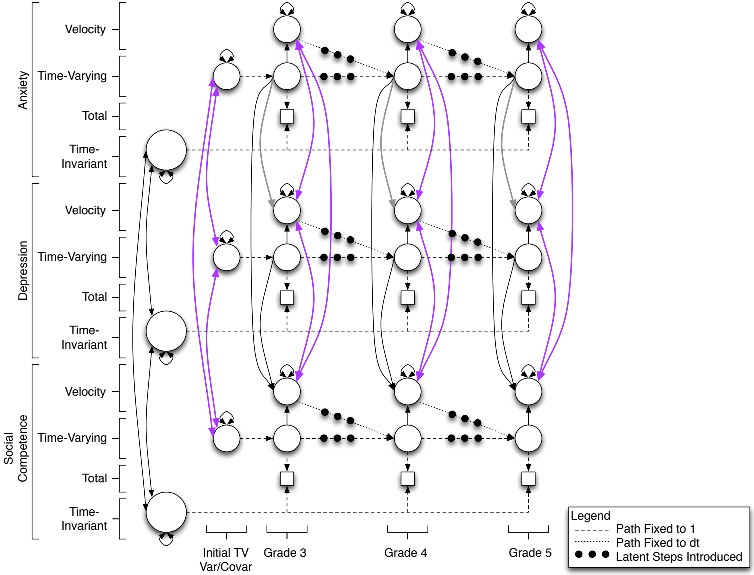


Continuous Time Model



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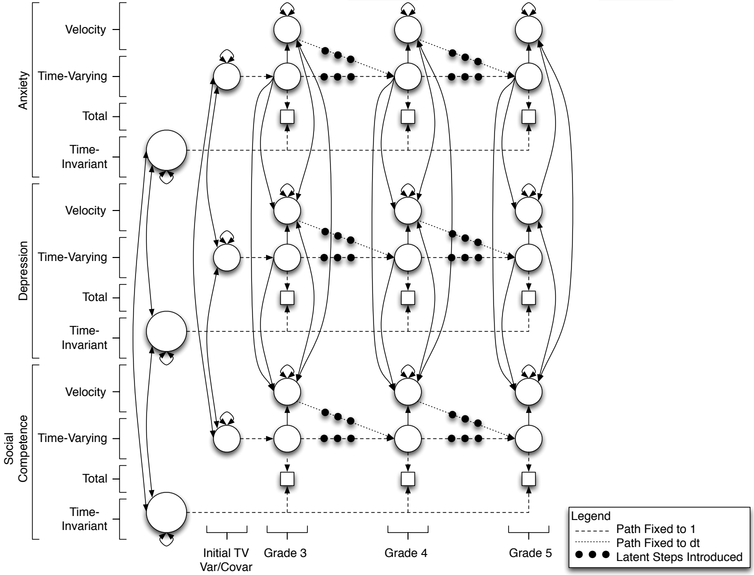
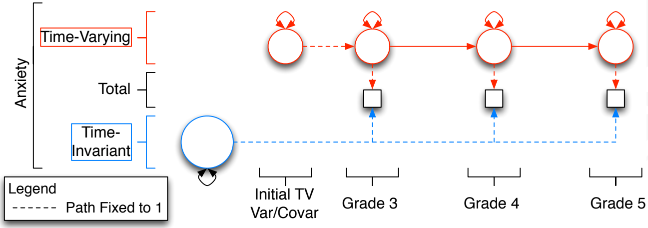
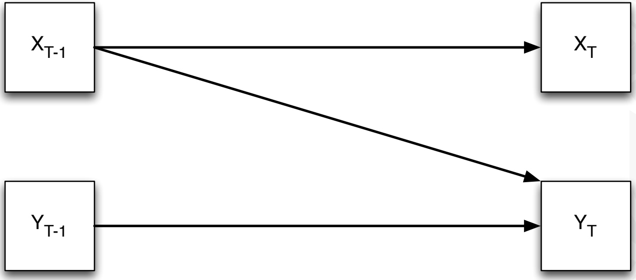


Substantive Results & Potential

- Model Fit Equivalent (same χ^2, df)
- Time-Invariant Effects
  - Means, Variances, Covariances are the same
- Time-Varying Effects



Time Varying Effects: Discrete Time



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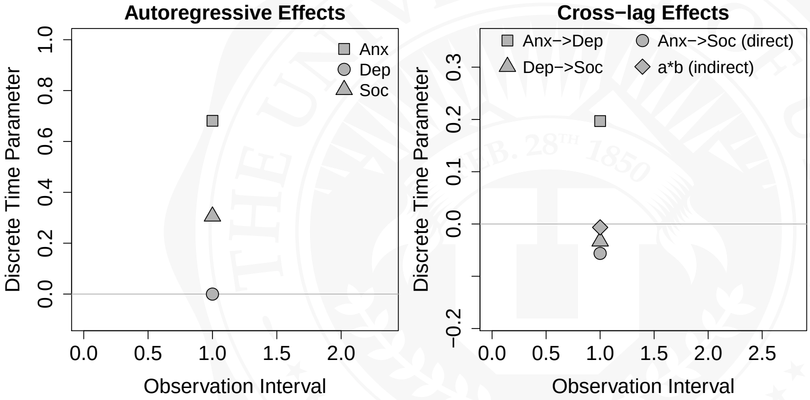
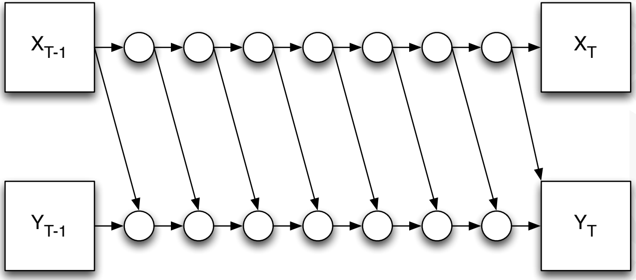


Time Varying Effects: Continuous Time



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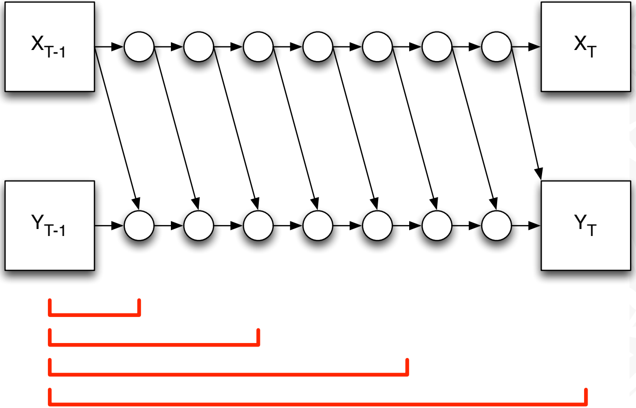
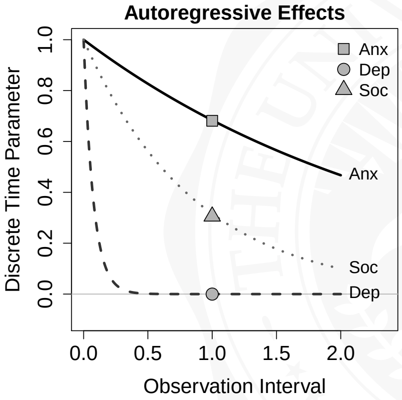
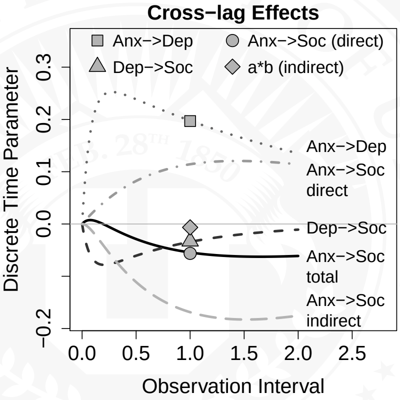
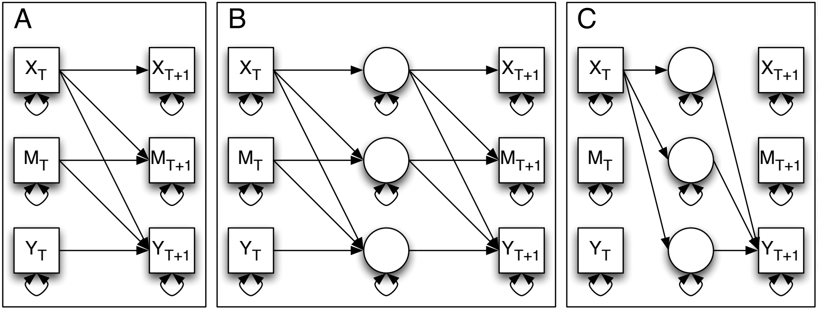


Time Varying Effects: Discrete & Continuous Time



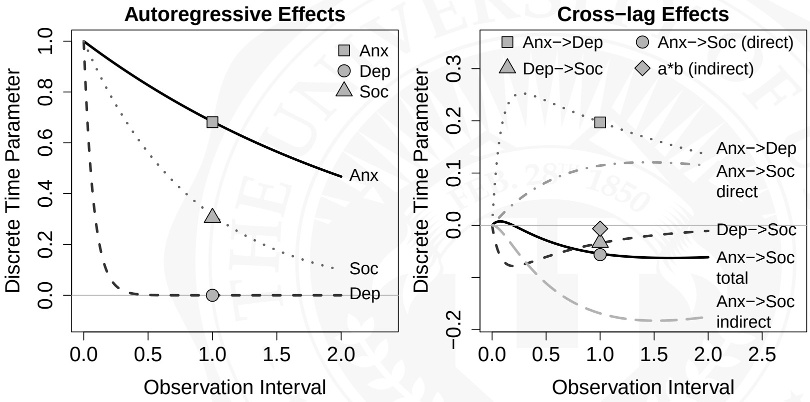


Direct & Indirect Effects: Changing Inferences



From a continuous time perspective, constructs continue to interact and accumulate, even between observations

Time Varying Effects: Discrete & Continuous Time



- Differing Perspectives on Modeling Time Affect Developmental Inferences

Challenges & Potential

- Discrete CLPM: Observations are regressed on prior observations without need for an explicit model
- Continuous Differential Equation: A specific model describing processes is required; strong theories of change processes

- Language of derivatives: Level, Velocity, Acceleration
- Coupling between derivatives
- Need for very specific theories of processes

Continuous Time Challenges

- Multiple methods have emerged, depending on the characteristics of the data (number/frequency of observation, assumptions about system dynamics)
    - Analytic Solutions to Stochastic Differential Equations (e.g., Exact Discrete Model, Ornstein–Uhlenbeck process)
    - Numerical Approximations (e.g., Integration of Structural-Differential Equations)
    - Generalized Local Linear Approximation, Generalized Orthogonal Derivative Estimates
    - Latent Differential Equations
    - Kalman Filter
    - Functional Data Analysis
- Software resource are available for some models, but not in all software packages
- Many pre-programmed software make it easy to run a model, but may not flexibly adapt to novel models

Differing Perspectives on Modeling Time

Discrete Time

- Many relations can be captured be correlating/regressing repeated observations; flexible
- Effects depend on lag between observations; "snapshot" of effects
- May misattribute some variance, depending on sampling rate and nature of how constructs change in time

Continuous Time

- Requires a specific model of change from one observation to the next; tests of specific processes
- Can compare differing change relations (level→velocity, velocity→velocity), even with few observations
- Effects can be estimates for a range of lags; may offer insight as to how effects change as a function of lag
- Accuracy of variance attribution likely depends on match between model and reality

- Deboeck, P. R., Preacher, K. J. & Cole, D. A. (in press). Mediation Modeling: Differing Perspectives on Time Alter Mediation Inferences. In K. van Montfort, J. Oud & M. Voelkle (Eds.) *Continuous Time Modeling in the Behavioral and Related Sciences*. Springer.
- Deboeck, P. R., Nicholson, J. S., Kouros, C., Little, T. D. & Garber, J. (2015). Integrating developmental theory and methodology: Using derivatives to articulate change theories, models, and inferences. *Applied Developmental Science, 19* (4), 217–231.
- Deboeck, P. R. & Boulton, A. J. (2016). Integration of Stochastic Differential Equations using Structural Equation Modeling: A Method to Facilitate Model Fitting and Pedagogy. *Structural Equation Modeling, 23*, 888–903.
- Deboeck, P. R. & Preacher, K. J. (2015). No Need to be Discrete: A Method for Continuous Time Mediation Analysis. *Structural Equation Modeling, 23*, 61–75.